

Dear Family,

The next Unit in your child's mathematics class this year is **Growing, Growing, Growing: Exponential Relationships**. This Unit focuses on exponential relationships, in which a quantity grows larger or smaller at a changing rate rather than at a constant rate.

▶ Unit Goals

Exponential relationships are often encountered in such fields as business and biology. When you invest money in an account that earns compound interest, you are dealing with *exponential growth*. You can see *exponential decay* in the way the body metabolizes medicine.

Your child has previously studied linear growth, in which a fixed amount is repeatedly added to a beginning quantity to produce a sequence of values. For example, in the sequence 2, 5, 8, 11, 14, . . . each term is 3 more than the previous term. Exponential growth involves patterns that are based on multiplication rather than addition. For example, in the sequence 3, 9, 27, 81, 243, . . . each term is 3 times the previous term.

The basic goal in *Growing, Growing, Growing* is for students to recognize situations, data patterns, and graphs that are modeled by exponential expressions and to use tables, graphs, and equations to answer important questions about exponential patterns. This Unit is designed to introduce the topic and to give students a sound, intuitive foundation on which to build later.

▶ Homework and Having Conversations About the Mathematics

You can help your child with homework and encourage sound mathematical habits during this Unit by asking questions such as:

- How can this relationship be detected in a table, graph, or equation? What is the growth factor?
- What table, graph, or equation would model the data?
- How could you answer questions about an exponential situation by studying a table, a graph, or an equation of the exponential relationship?
- How does this relationship compare to other relationships you have studied?
- Is the relationship between variables an example of exponential growth or decay? Explain.

You can help your child with his or her work for this Unit in several ways:

- Talk with your child about the applications that are presented in the Unit and similar applications that you encounter in your daily activities.
- Discuss saving practices in your household. You might investigate with your child how investments, mortgages, or insurance policies involve exponential growth.

In your child's notebook, you can find worked-out examples, notes on the mathematics of the Unit, and descriptions of the vocabulary words.

▶ Common Core State Standards

While all of the Standards for Mathematical Practice are developed and used by students throughout the curriculum, *making sense of problems and persevering in solving them* as well as *reasoning abstractly and quantitatively* are used repeatedly as students compare relationships to one another. *Expressions and Equations* and *Functions* are two of the domains focused on as students build and interpret functions and work with exponents.

A few important mathematical ideas that your child will learn in *Growing, Growing, Growing* are on the next page. As always, if you have any questions or concerns about this Unit or your child's progress in the class, please feel free to call.

Sincerely,

Important Concepts

Exponential Growth

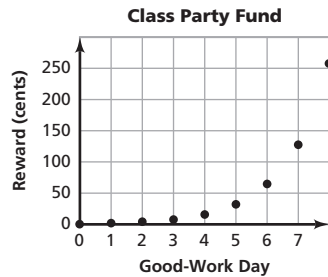
An exponential pattern of change can often be recognized in a verbal description of a situation or in the pattern of change in a table of (x, y) values.

The exponential growth in rewards for good-work days in the example can be represented in a graph. The increasing rate of growth is reflected in the upward curve of the plotted points.

Examples

Suppose a reward is offered for days of good work. At the start, 1¢ is put in a party fund. On the first good-work day, 2¢ is added; on the second good-work day, 4¢ is added; and on each succeeding good-work day, the reward is doubled. How much money is added on the eighth good-work day?

Good-Work Day	Reward (cents)
0 (start)	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	■



Growth Factor

A constant factor can be obtained by dividing each successive y -value by the previous y -value. This ratio is called the *growth factor* of the pattern.

For each good-work day, the reward doubles. You multiply the previous award by 2 to get the new reward. This constant factor can also be obtained by dividing successive y -values: $\frac{2}{1} = 2$, $\frac{4}{2} = 2$, etc.

Exponential Equation

Examining the growth pattern leads to a generalization that can be expressed as an equation.

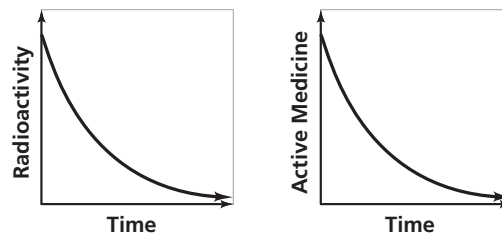
An exponential growth pattern $y = a(b)^x$ may increase slowly at first but grows at an increasing rate because its growth is multiplicative. The growth factor is b .

Day	Calculation	Reward (cents)
0	1	1
1	$1 \times 2 = 2^1$	2
2	$1 \times 2 \times 2 = 2^2$	4
3	$1 \times 2 \times 2 \times 2 = 2^3$	8
⋮	⋮	⋮
6	$1 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6$	64
⋮	⋮	⋮
n	$1 \times 2 \times 2 \times \dots \times 2 = 2^n$	2^n

On the n th day, the reward R will be $R = 1 \times 2^n$. Because the independent variable in this pattern appears as an exponent, the growth pattern is called exponential. The growth factor is the *base* 2. The *exponent* n tells the number of times the 2 is a factor.

Exponential Decay

Exponential models describe patterns in which the value decreases. Decay factors result in decreasing relationships because they are less than 1.



$$y = 50\left(\frac{1}{2}\right)^n$$

Rules of Exponents

Students begin to develop understanding for the rules of exponents by examining patterns in a powers table for the first 10 whole numbers.

By examining the multiplicative structure of the bases:

$$8^2 = (2 \times 2 \times 2)^2 = (2^3)^2 = 2^6; \text{ the general pattern is } (b^m)^n = b^{mn}$$

$$9 \times 27 = 243 \text{ or } 3^2 \times 3^3 = 3^5; \text{ in general, } (b^m)(b^n) = b^{m+n}$$

$$4 \times 25 = 2^2 \times 5^2 = (2 \times 5)^2 = 10^2 = 100; \text{ in general, } (a^m b^m) = (ab)^m$$

Similar explorations lead to the rule $\frac{a^m}{a^n} = a^{m-n}$.